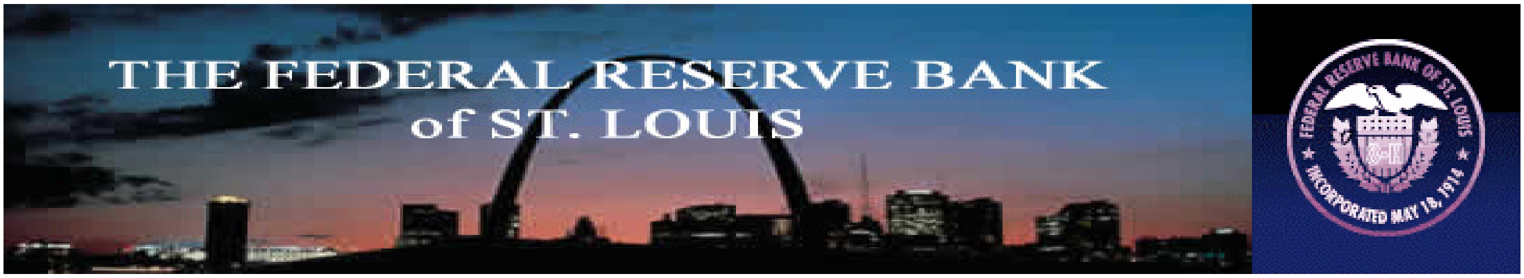

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Uniform Working Hours and Structural Unemployment

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Abstract

In this paper, we construct a simple model based on heterogeneity in workers' productivity and homogeneity in their working schedules. This simple model can generate unemployment, even if wages adjust instantaneously, firms are perfectly competitive, and firms can perfectly observe workers' productivity and effort. In our model, it is optimal for low-skilled workers to be unemployed because, on the one hand, firms do not find it optimal to hire low-skilled workers when labor hours must be synchronized across heterogeneous workers, and on the other hand, low-skilled workers do not find it attractive working for the same hours as high-skilled workers at competitive wages based on productivity. Thus our model offers an alternative explanation for why unskilled workers are a primary source of structural unemployment. (*JEL classification*: E0, J6.)

Keywords: Unemployment; Structural Unemployment; Equilibrium Unemployment; Synchronized Working Hours; Uniform Working Hours; Heterogeneous Labor; Indivisible Labor.

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1 Introduction

Unskilled workers are a primary source of structural unemployment. It is well documented that the unemployment rate of unskilled workers is much higher and more sensitive to the business cycle than that of skilled workers (e.g., Nickell and Bell, 1996; Bowlus et al., 2001).[1]

The conventional wisdom regarding structural unemployment explains it as wage rigidity (e.g., the efficiency wage theory). But no satisfactory explanations exist in this literature for why unskilled workers are more likely to be unemployed than skilled workers. Existing studies generally attribute the difference in unemployment rate between the skilled and unskilled workers to a weaker labor demand of unskilled workers. This interpretation, however, depends implicitly on the assumption that wages for unskilled workers are stickier than wages for skilled workers, or that the efficiency wage premium is higher for low-skilled workers than it is for skilled workers. Few empirical studies, however, have been carried out to test or to support this assumption.

Standard textbook explanations for why unskilled workers are more likely to contribute to structural unemployment are rarely available, and if available, they also tend to be very vague and not compelling. A typical statement in this regard can be found in Abel and Bernanke (2001, p95):

> "...unskilled or low-skilled workers often are unable to obtain desirable, long-term jobs. The jobs available to them typically offer relatively low wages and little chance for training or advancement. Most directly related to the issue of

[1] Also, using the 1993 U.S. Current Population Survey March Supplement, the unemployment rate for prime age male without finishing high school is 13.86%, while the unemployment rate is only 6.24% for those who graduated from high schools.

> structural unemployment is the fact that jobs held by low-skilled workers often don't last long. After a few months the job may end, or the worker may quit or be fired, thus entering another spell of unemployment.... Because of factors such as inadequate education, discrimination, and language barriers, some unskilled workers never make the transition to long-term employment and remain chronically unemployed."

The implicit explanation in the quoted passage is that low-skilled workers can only find short-term jobs, since long-term jobs require skills or specific human capital. According to this explanation, however, unemployment due to the lack of skills should be characterized not as structural but frictional unemployment, because if it is true that short-term jobs end more quickly and more frequently than long-term jobs do, then the major reason for unskilled workers to be unemployed is that they are forced more frequently to enter the job search process, contributing to frictional unemployment. This is obviously not the conclusion the the above message intends to reach, as chronic unemployment due to lack of skills is different from frictional unemployment due to search. This leaves the quoted passage with the only logical implication for the higher rates of unemployment of unskilled workers: there are fewer short-term jobs available than long-term jobs.

Thus, existing studies and conventional wisdom alike generally attribute, in one way or another, implicitly or explicitly, the difference in the unemployment rates between the skilled and unskilled workers to the weaker labor demand of unskilled workers, without offering an explicit explanation as to why a lower demand for labor necessarily leads to a higher rate of unemployment. Unless wages are stickier (or the efficiency wage premium is higher) for the unskilled workers, equilibrium in the labor market always equates supply and demand. Hence a lower labor demand or a lower wage rate does not by itself explain a higher rate of unemployment.

In this paper, we offer a simple model to explain the aforementioned facts without resorting to wage stickiness or the efficient wage theory. The core of our explanation is based on the stylized fact that working hours of both skilled and low-skilled workers are highly synchronized. For example, managers, secretaries, technicians, and workers all work during the same hours in a day and during the same days in a week (e.g., from 8:00 am to 5:00 PM in a day and from Monday to Friday in a week).[2]

Costa (2000) has documented that the distribution of daily working hours are highly compressed. For men aged 25-64, the difference between the 90th percentile and the 10th percentile of the daily working hours distribution is only 2 hours in both 1973 and 1991. Moreover, she also finds that the daily working hours of median workers are the same as those of workers at the 10th percentile in the distribution. Using the most recent 1999 US. Current Population Survey (CPS) March Supplement file, we find that only 8.45% of the prime age (24-64) males worked less than 8 hours per day while more than 91% worked 8 hours per day or longer.

There is no doubt that the synchronization of working hours is due at least partly to biological reasons. For example, it is only natural for people to sleep at night and work during daytime. Hence, working for 4 hours in the morning and 4 hours in the afternoon with a lunch break in the middle appears to be a natural arrangement. However, there also

[2] Costa (2000, p160) claims that the most common pattern of work is to begin at 8 A.M. and end at 5 P.M. from Monday to Friday.

exist important economic reasons for adopting a uniform working schedule. For example, accomplishing a task requires the coordination of many workers of different skill levels during the same period of time (think of the operation of an assembly line). Such arrangement reduces not only coordination costs but also many other sorts of fixed production costs (e.g., management costs, utility bill costs and other types of costs associated with operation of capital). This interpretation is consistent with Costa's (2000, p178) arguments. She claims that the egalitarian hours distribution is the result of coordination of work activities within and across firms. Moreover, Costa also points out that the synchronization of leisure-time activities might also be the reason for the compression of daily working hours distribution.

When workers differ in their skill levels (productivity), highly synchronized working time has an important consequence: it creates unemployment. And it turns out that it is the low-skilled workers who are more likely to be unemployed than skilled workers in a competitive labor market with synchronized working hours.

The intuition is as follows. Due to the heterogeneity of skills, competitive wage rates (measured by workers' marginal productivity) differ across workers. Suppose all workers share the same propensity to work. They will then opt to supply a different number of hours in response to different wage rates, with low-skilled workers working for fewer hours and high-skilled workers working for longer hours. The synchronization of the working schedule, however, requires that all types of workers work for the same length of time regardless of skills, say 8 hours per day or 40 hours per week. Low-skilled workers may therefore find the required working hours far longer than preferred at the competitive wage rates measured by marginal product. On the other hand, it is not in the firm's interest to pay the low-skilled workers at a wage rate above their marginal product in order to entice them to work for longer hours than they prefer. Consequently, unemployment will fall upon low-skilled workers, and only low-skilled workers are willing to accept part-time jobs.[3]

A synchronized working scheme thus creates a dilemma: low-skilled workers would choose to work if the wages were high enough to match their utility cost, which few firms would like to offer since they are above the workers' marginal products; or they could work for fewer hours at the market determined wages, which is difficult for firms, however, due to the synchronization of working schedules among workers.[4] Our theory thus predicts that there exists a natural rate of structural unemployment due to synchronization of working hours, and that part-time jobs of various duration, if available, are more likely to be occupied by unskilled workers.[5] Also, it is just a natural consequence of our theory, without resorting to the notions of sticky wages or efficiency wages, that lower labor demand due to lower labor

[3] Workers can also differ in propensity to work (i.e., preferences). Similar arguments can show that workers with lower propensity to work, given the same skill levels, will be more likely to become unemployed under the scheme of synchronized working hours. Thus synchronized working hours can create unemployment as long as people differ, either in terms of propensity to work or in terms of skills of labor. To explain the phenomenon that the rate of unemployment is higher for low-skilled workers than for high-skilled workers, we assume that the variation in propensity to work is smaller than the variation in productivity or skills, although this does not rule out the possibility that some workers are unemployed because of low propensity to work.

[4] Even part-time jobs require synchronized working hours among the part-time workers. Hence the dilemma does not go away completely by creating part-time jobs unless firms are capable of creating a whole spectrum (continuum) of jobs with all possible lengths of working hours, each for one specific worker with a particular level of skills. This, however, is obviously too costly for firms to implement since it undoes the benefit of synchronization - to exploit the complementarity of labor and to reduce the cost of coordination among workers of different skills. This is perhaps why part-time jobs are not as common as full-time jobs in manufacturing industries where the degree of labor complementarity and the coordination costs of labor are high. And this is perhaps also the reason we rarely observe part-time jobs with arbitrary length, except in the service sector where the intensity of capital service and the complementarity of labor are low (which implies that the coordination cost is low).

[5] The 1992 CPS data based on males of age 25-55 shows that in the year of 1991, among part-time workers, 24.25% are high school dropouts whereas among full-time workers that number is only 12.59%. The same data also shows that among high school dropouts, 11.64% work as part-time workers whereas among high school graduates that number is merely 5.32%.

productivity is intrinsically associated with higher rates of unemployment. Our theory thus has an important policy implication: a simple solution for reducing structural unemployment is to create more part-time jobs with flexible length of working hours. This solution is feasible, however, only if coordination costs among different workers can be reduced.[6]

The arguments presented in the paper are akin to the theory of indivisible labor (Hansen, 1985, and Rogerson, 1988). According to that theory, unemployment arises because people can only choose to either work or not to work. Hence in equilibrium some individuals may be unemployed. This theory, however, cannot explain the stylized fact that both the number of employed people and the number of working hours are variable and highly volatile during business cycles. In addition, it cannot explain why it is the low-skilled workers who are more likely to be unemployed. In our model, both the rate of employment and the length of synchronized working hours can vary in response to aggregate disturbances, and it is the low skilled workers who are most sensitive to the business cycle. In other words, equilibrium unemployment exists in our model not because of *ex ante* indivisibility of labor – in fact working hours in our model are infinitely divisible, but because of the synchronization of working hours across heterogenous workers that gives rise *ex post* to a rigidity in the labor market similar to that of indivisible labor. Hence the theory provided in this paper can be viewed as a natural extension of the indivisible labor theory.

The remainder of this paper is organized as follows. Section 2 sets up the model. Section

[6] This explains why the service sector where the coordination cost is low due to low capital intensity is an important sector for absorbing low skilled labor and for creating part-time jobs. With the development of IT technology, which reduces coordination costs, more and more people can choose to work at home. This should also prove useful in reducing structural unemployment. But this trend is so far associated only with high-skilled workers, since operating computers at home requires skills.

3 proves the existence of equilibrium and derives the equilibrium unemployment rate – the "natural rate". A calibrated numerical example is given in Section 4. The case of indivisible labor is discussed in Section 5, and section 6 concludes.

2 The Model

There is a continuum of agents distributed in the interval $i \in [0, 1]$, working for a representative firm (say, a pin factory). They have identical propensity to work but differ in their skills. Let p_i denote individual $i's$ skill level (productivity), which is non-negative and is decreasing in i:

$$\frac{dp_i}{di} < 0. \tag{1}$$

If worker i gets to supply n_i hours of labor, her contribution to output (intermediate goods) is measured by a diminishing returns to scale technology:

$$y_i = p_i f(n_i), \quad f' > 0, f'' \leq 0. \tag{2}$$

However, we assume that

1) The production technology is accessible to a worker *only* when the factory is open. That is, there is some time when a factory is closed, and hence inaccessible to labor. Output can only be generated when the factory is open, so even hired labor is unproductive unless it is present when the factory is open.

2) Workers are substitutable *ex ante* but become complementary *ex post*. That is, once a worker is hired, her labor input is essential for the production of not only her own output, but also of everybody else's output in the factory. In other words, labor of different skills are complementary to each other at the work place (imagine workers being assigned to different

positions on an assembling line), so that labor is productive *only* when all other employed workers are present simultaneously during the factory's open hours.

These assumptions suggest that we can rewrite the production function as a Leontief type technology:

$$y_i = p_i f(\min\{n_0, ..., n_j, ..., n_I, N\});$$

where N is the factory's operation time, which can also be interpreted as capital's working hours, and $I \in [0, 1]$ is the cut-off skill level hired by the firm, which can also be interpreted as the employment rate. This production function implies that a worker's labor productivity is zero if the factory does not open (assumption 1). In addition, it implies that a worker's marginal product is zero if her working hours exceed the minimum working hours of other employed workers (assumption 2). Hence, as in Adam Smith's pin factory, coordination of labor of different skills and synchronization of working hours are essential for production.[7]

Let the final output of the firm be an aggregation of output produced by all employees:[8]

$$Y = \int_{i=0}^{I} y_i di = \int_{i=0}^{I} p_i f(\min\{n_j, N\}) di, \quad j \in [0, I]. \tag{3}$$

Both the rate of employment (I) and the factory's hours of operation (N) are determined by the firm in order to maximize profit.

[7] The Leontief technology assumption is quite extreme and it is made here for simplicity only. Basically, what we want to show is that as long as there are benefits associated with labor synchronization, then the demand function for labor for each individual will be determined by the operation time of capital, N. Thus, some workers must be unemployed if the optimal length of working hours determined by the firm exceeds what the workers prefer.

[8] Note that since we assume that the complementarity of labor among workers takes place only at the working site, hence unemployed workers do not affect employed workers' labor productivity because unemployed workers do not take position on the assembly line.

Cost minimization by the firm implies a perfect synchronization of working hours across employees. Hence the demand for labor for each worker i is given by:

$$n_i = N, \qquad \text{for all } i \leq I; \tag{4}$$

where $\{N, I\}$ remain to be determined in equilibrium. Demand for labor follows such a simple rule because working hours for each individual longer than others'have zero marginal product and working hours shorter than others'would render any extra labor of the rest of the employees unproductive. Thus, worker i's effective production function becomes:

$$y_i = p_i f(N),$$

where $f()$ is differentiable in N. The representative firm's profit function then takes a simple form,

$$\Pi = \int_{i=0}^{I} [p_i f(N) - w_i N] \, di, \tag{5}$$

where Π denotes profit. We assume that the competitive real wage received by each employed worker is determined by a simple marginal-product rule:[9]

$$w_i = p_i f'(N). \tag{6}$$

Given that all workers have the same upward-sloping labor supply function, $n^s(w)$, we may denote the reservation wage for all types of workers as $\bar{w}(N)$. Hence, type i individual will accept a job and become employed if $p_i f'(N) \geq \bar{w}(N)$, and will not accept the job and

[9] Note that the externality among workers appears to give workers the power to bargain for shares of output. However, since we assume that a worker has access to the production technology only after she is hired, it is therefore the firm that has the power to internalize and exploit the externality, provided that firms are able to replace immediately any workers who quit, which we assume in the paper.

hence become unemployed if $p_i f'(N) < \bar{w}(N)$.[10]

Figures 1 and 2 illustrate the point. The upward sloping line in figure 1 represents the labor supply curve that is assumed to be the same across all agents $i \in [0,1]$. The downward sloping lines represent labor demand curves for agents with different productivity levels. The demand for labor is obviously weaker for lower-productivity workers at any given wage rates. But this alone does not necessarily give rise to unemployment. Notice that if working hours are not required to be synchronized across agents, competitive equilibrium then implies that all agents are employed regardless of their skill levels. In such an equilibrium each worker has her own specific length of working hours, and workers differ only in the length of working hours and in wage rates, not in their employment status. In figure 1, for example, agent i_0 works for n_0 hours, agent I works for N hours, and agent i_1 works for n_1 hours, etc.

Coordination costs and complementarity of labor, however, make it extremely costly for firms to offer a complete spectrum of working hours according to each individual's productivity. Cost minimization requires synchronization of labor, implying that workers cannot work for any arbitrary hours of their desired labor supply at the competitive wage rates based on their productivity. They must either work for the same length of time as the others do during the factory's operation hours or not work at all. Consequently, there may exist

[10] In order to simplify our analyses without loss of generality, we assume that the reservation wage is determined by the inverse labor supply function $w(N)$, implying that staying unemployed receives higher utility than working for hours longer than preferred. This assumption can be easily relaxed using explicit utility maximization without affecting the major conclusions reached (see section 5 for such an analysis). For the same purpose, we also assume that "part-time" jobs with hours shorter than N are not available in the model. It is straightforward to extend the model to allow for part-time working hours. The major insight of our theory continues to hold as long as there also exists synchronization in labor hours for the part-time jobs. In reality, although part-time jobs do exist, they are not all set for arbitrary lengths of time. Namely, there still exists synchronized minimum hours during which people are required to work for in part-time jobs unless capital and coordination are not required.

unemployment. Figure 2, for example, shows that agent i_0 is unemployed because her labor supply curve intersects with her labor demand curve at a location that is below the uniform working hours N. At the uniform working hours (N), agent $i_0's$ utility cost is W, which exceeds the competitive real wage she receives (i.e., W_0). In fact, all workers with skills close to type i_0 agent can afford only "part-time" jobs (with hours less than N) at the competitive wage rates measured by their marginal product of labor, although they are certainly interested in (or looking for) "full-time" jobs that can pay them wages that match their disutility of working. Hence they satisfy the definition of structural unemployment given by the literature.

In figure 2, only type I or type i_1 agents are employed, where I is also the optimal rate of employment to be determined by the profit-seeking firm along with the synchronized working hours N. Competitive real wages paid to employed workers obviously differ across workers' types due to heterogeneity in productivity. Some of them (say agent i_1) may find the wage rates (e.g., W_1) so attractive (as it is far above their marginal disutility of working) that they are willing to supply hours much longer than N but can nevertheless work only for N hours. In fact, all employed workers except the cut-off type (I) work for wage rates above their labor supply curve. Note that wages paid to employed workers are also higher than their respective market-clearing levels. For example, agent i_1 receives real wage W_1 from the representative firm while her market-clearing real wage (determined by equating supply and demand of labor with respect to type i_1 agents) is between W_1 and W. The cut-off agent I is the only exception, with her received real wage just equal to the market-clearing level (W).

This feature of the model that received wages are above market clearing wages is reminiscent of the efficiency-wage literature (see Yellen, 1984, Katz, 1986, and Akerlof and Yellen, 1986 for surveys and references), although arising for an entirely different reason. In our model, equilibrium wage rates being higher than market clearing wages for high-skilled workers is purely because of the synchronization of working hours, not because of any incentive problems due to unobservable work effort or productivity. Similarly, the fact that low-skilled workers are unemployed in our model is not because of weaker demand for low-skilled labor *per se*, but because of the synchronization in working hours that results in wages paid to low-skilled workers (e.g., W_0) being below their least acceptable levels (i.e., W).

Whether unemployment in this model is "voluntary" or "involuntary", therefore, depends purely on the point of view. It is "voluntary" in the sense that low-productivity workers (such as those represented by i_0) refuse to take a job working for N hours and being paid at the competitive wage, W_0, which is below their disutility of working (W). It is "involuntary", on the other hand, in the sense that firms refuse to hire the low-skilled workers (such as i_0) according to their reservation wages (such as W) or to any perceived market prevailing wages.[11]

3 Equilibrium

Equilibrium is defined as a pair of synchronized working hours and the rate of employment, $\{N^*, I^*\}$, that maximizes firm's profit. In this section, we prove the existence of equilibrium and we conduct comparative statics with respect to changes in technology parameters.

[11] By definition, a worker is said to be "involuntarily unemployed" if she is willing to work at the market-prevailing wage but cannot find a job. In our model, the market-prevailing wages are the wage rates paid to the employed workers (e.g., anywhere between W and W_1). Since the market-prevailing wage rates are above the unemployed workers' marginal products, these workers cannot find jobs.

Proposition 1 *If the supply of labor is an increasing function of the real wage (upward sloping), then for any given uniform working hours $N > 0$, there exists a cut-off point $I(N)$, such that worker i is unemployed if $i > I(N)$, and employed if $i \leq I(N)$.*

Proof. Let $w^s(n)$ be the inverse labor supply function of type i worker (the reservation wage). Since the labor supply of the cut-off type I is exactly the same as her labor demand, we have:

$$p_I f'(N) = w^s(N). \tag{7}$$

Equation (7) determines the cut-off type's productivity as a function of the synchronized working hours N :

$$p_I = w^s(N)/f'(N), \quad f'(N) > 0. \tag{8}$$

Since the index function p_I is a function of I, the cut-off point $I(N)$ is therefore implicitly determined. ■

For any employed worker with $i < I(N)$, her real wage exceeds her reservation wage:

$$p_i f'(N) > w^s(N), \tag{9}$$

and her equilibrium labor supply is given by $n_i = N$. For any unemployed worker with $i > I(N)$, her equilibrium labor supply is zero.

The cut-off point, $I(N)$, measures the rate of employment given N. It is a decreasing function of the synchronized working hours N since equation (8) implies that the cut-off worker's productivity p is increasing in N. The intuition is that only higher-productivity workers are willing to work for longer hours at the competitive wage rates, consequently less people are attracted to work as the working hours increase.

Proposition 2 *Define the cumulative product index as* $P(N) \equiv \int_{i=0}^{I(N)} p_i di \geq 0$, *the elasticity of the cumulative product with respect to the factory operation time* N *as* $\varepsilon(N) \equiv \frac{P'(N)N}{P(N)}$, *and the elasticity of a worker's output with respect to hours,* $\alpha \equiv \frac{f'(N)N}{f(N)}$. *Assume that* α *is constant. An optimal synchronization time* N^* *exists and is determined by the condition:*

$$-\varepsilon(N) = \alpha. \tag{10}$$

Proof. The firm's optimization program is to choose a uniform working hours N to solve:

$$\max_N \ \Pi = \int_{i=0}^{I(N)} [p_i f(N) - w_i N]\, di = (1-\alpha) f(N) \int_{i=0}^{I(N)} p_i di, \tag{11}$$

where w_i is defined by (6), the cut-off point $I(N)$ is determined by (8), and $\alpha \in (0, 1]$ is the constant output elasticity of hours.[12] Using the definition for the cumulative productivity index, P, the profit maximization program can then be rewritten as

$$\max_N \Pi = (1-\alpha) f(N) P(N). \tag{12}$$

Without loss of generality, assume $f(N) = 0$ for $N = 0$, and $I(N) = 0$ for $N \geq M < \infty$. Since the profit function is non-negative over the domain $N \in R^+$ and it takes zero values at the two points, $N = \{0, M\}$, a maximum therefore exists in the open interval $N \in (0, M)$. This proves the existence of N^*. The necessary condition for optima is given by

$$f'(N)P(N) + f(N)P'(N) = 0, \tag{13}$$

which implies

$$\frac{f'(N)N}{f(N)} = -\frac{P'(N)N}{P(N)}, \tag{14}$$

or $\alpha = -\varepsilon(N)$.■

[12] When the production function is linear ($\alpha = 1$), the objective function can be defined as maximizing total revenue rather than total profit.

This optimal condition says that, given that the firm must choose a uniform working hours (N) across all types of agents with different skill levels, N should be chosen at the point where the elasticity of cumulative product with respect to hours (the percentage loss of aggregate output due to the loss of the number of employees as working hours increase) is equal to the elasticity of individual output (the percentage gain in individual's production as working hours increase).

To understand this condition, notice that the profit function, $\Pi = (1-\alpha)f(N)P(N)$, is a combination of output due to per-worker quantity ($f(N)$) and an index of aggregate quantity (P) of all employees. The quantity per worker increases with hours worked per person ($f'(N) > 0$). The aggregate quantity of all employees (P), however, decreases with hours worked per person because longer uniform working hours imply that fewer workers are employed under competitive real wages, hence the aggregate product index, $P = \int_{i=0}^{I} p_i di$, decreases. (Note $P'(N) = P'(I)I'(N) < 0$ since $I'(N) < 0$). Increasing working hours in the factory thus has two opposite effects on total profit:

$$\frac{d\Pi}{dN} = f'(N)P(N) + f(N)P'(N), \tag{15}$$

where the first term measures the marginal gain given the number of employees, and the second term measures the marginal loss due to a reduction in the rate of employment as working hours increase.

Proposition 3 *If* $\frac{P''N}{P'} > -(1+\alpha)$, *then the equilibrium is unique.*

Proof. Differentiating equation (13) again gives

$$\frac{d^2\Pi}{dNdN} \quad = \quad f''P + f'P' + f'P' + fP'' \tag{16}$$

$$= \quad f'P'\left[\frac{f''N}{f'}\frac{P}{P'N} + 2 + \frac{f}{f'N}\frac{P''N}{P'}\right].$$

Note that $\frac{f'N}{f} = \alpha$, $\frac{f''N}{f'} = \alpha - 1$, $\frac{P'N}{P} = \varepsilon = -\alpha$, and $P' < 0$. Hence,

$$\frac{d^2\Pi}{dNdN} = f'P'\left[\frac{1-\alpha}{\alpha} + 2 + \frac{1}{\alpha}\frac{P''N}{P'}\right] < 0 \tag{17}$$

if and only if $\frac{P''N}{P'} > -(1+\alpha)$. ■

The intuition for the condition, $\frac{P''N}{P'} > -(1+\alpha)$, is that we require that the loss of cumulative product due to the loss of low-skilled workers caused by an increase in the uniform working hours does not accelerate too fast when N increases, meaning that the cut-off function I does not decrease too fast as N increases, or that the inverse labor supply curve in figure 1 is not too steep. Suppose that the condition fails to hold, e.g., the inverse labor supply curve in figure 1 is vertical, then we can imagine multiple or even a continuum of equilibria for the cut-off function I.

The optimal rate of employment is then given by $I(N^*)$, in which N^* solve equation (10). A "natural" rate of unemployment in the economy can then be defined as

$$U^{NR} = 1 - I(N^*), \tag{18}$$

which depends on both preferences and technology parameters.

The following two propositions establish the direction of changes in both I and N when technology parameters change. For that purpose, we introduce an aggregate technology shifter A into the workers' production function:

$$y_i = Ap_i f(N), \tag{19}$$

so that the cut-off condition becomes:

$$p_I = \frac{w^s(N)}{Af'(N)}. \tag{20}$$

Note that the cut-off condition implies that I is decreasing in N and that $\frac{\partial I}{\partial A} > 0$ holding N constant (since $\frac{dp_i}{di} < 0$).

Proposition 4 $\frac{dN}{dA} > 0$ *if* $\frac{\partial^2 P}{\partial I^2} \leq 0$. *Namely, the response of N to changes in the aggregate technology level is positive if the cumulative product index P is non-convex in I.*

Proof. Rewrite the first-order condition (10) as

$$\alpha P(N, A) = -P_N(N, A)N(A). \tag{21}$$

Totally differentiating both sides of the equation with respect to A gives

$$\alpha \left[P_N \frac{dN}{dA} + P_A \right] = -\left[P_{NN} N \frac{dN}{dA} + P_{NA} N + P_N \frac{dN}{dA} \right]. \tag{22}$$

Collecting terms gives

$$\alpha P_A + P_{NA} N = -P_N \left[\frac{P_{NN} N}{P_N} + (1+\alpha) \right] \frac{dN}{dA}. \tag{23}$$

Note that the uniqueness of the equilibrium requires $\left[\frac{P_{NN} N}{P_N} + (1+\alpha) \right] > 0$ (Proposition 3). We also know that $P_N < 0$ since $P_N = -\alpha \frac{P}{N}$ as in (21). Hence,

$$\frac{dN}{dA} > 0 \quad \text{if} \quad \alpha P_A + P_{NA} N > 0. \tag{24}$$

But we know that $P_A = \frac{\partial P}{\partial I} \frac{\partial I}{\partial A} > 0$, where $\frac{\partial P}{\partial I} > 0$ and $\frac{\partial I}{\partial A} > 0$ since the cut-off worker's productivity p in (20) is decreasing in A holding N constant. Therefore, it suffices to require only

$$P_{NA} > 0, \tag{25}$$

where P_{NA} satisfies

$$P_{NA} = \frac{\partial}{\partial A}\left(\frac{\partial P}{\partial N}\right) = \frac{\partial}{\partial A}\left(\frac{\partial P}{\partial I}\frac{\partial I}{\partial N}\right) = \frac{\partial^2 P}{\partial I^2}\frac{\partial I}{\partial A}\frac{\partial I}{\partial N} + \frac{\partial P}{\partial I}\frac{\partial^2 I}{\partial N \partial A}. \tag{26}$$

Denote these two terms as $P_{II}I_A I_N + P_I I_{NA}$, in which we know $I_A I_N < 0$ since $\frac{\partial I}{\partial A} > 0$ and $\frac{\partial I}{\partial N} < 0$; and we also know $P_I I_{NA} > 0$ since $\frac{\partial P}{\partial I} > 0$ and sign$\left(\frac{\partial^2 I}{\partial N \partial A}\right)$ = -sing$\left(\frac{\partial^2 p}{\partial N \partial A}\right) = +$, where p is the cut-off worker's productivity satisfying $\frac{\partial^2 p}{\partial N \partial A} < 0$ (see equation 20). Therefore, $P_{NA} = P_{II}I_A I_N + P_I I_{NA} > 0$ if $P_{II} \leq 0$. ■

This proposition is intuitive since a higher A raises each worker's productivity. However, if the second-order condition, $P_{II} \leq 0$, is not satisfied, then it is possible for N to decrease in response to an increase in A, because in that case the firm opts to increase the number of workers (I) by so much that it becomes optimal for firms to reduce operating hours N.

Proposition 5 $\frac{dI}{dA} > 0$ *if the elasticity of equilibrium hours (N) with respect to A satisfies*

$$\frac{dN}{dA}\frac{A}{N} < \frac{1}{\varepsilon_w + 1 - \alpha}, \tag{27}$$

where $\varepsilon_w > 0$ is the wage elasticity of labor supply.

Proof. Totally differentiating the cut-off condition (20) with respect to A yields

$$\begin{aligned} \frac{dp}{dI}\frac{dI}{dA} &= \frac{w'_N A f'_n \frac{dN}{dA} - w\left(f'_N + A f''_{NN}\frac{dN}{dA}\right)}{\left(Af'_N\right)^2} \\ &= \frac{wAf'_N\left[\frac{w'_N}{w} - \frac{f''_{NN}}{f'_N}\right]\frac{dN}{dA} - wf'_N}{\left(Af_N\right)^2}. \end{aligned} \tag{28}$$

Since $\frac{dp}{dI} < 0$, the requirement $\frac{dI}{dA} > 0$ implies that the right-hand side of the equation must be negative:

$$wAf'_N\left[\frac{w'_N}{w} - \frac{f''_{NN}}{f'_N}\right]\frac{dN}{dA} - wf'_N < 0, \tag{29}$$

which implies

$$\left[\frac{w'_N N}{w} - \frac{f''_{NN} N}{f'_N}\right] \frac{dN}{dA}\frac{A}{N} < 1, \tag{30}$$

or $\frac{dN}{dA}\frac{A}{N} < \frac{1}{\varepsilon_w + 1 - \alpha}$ since $\frac{f'' N}{f'} = \alpha - 1$. ■

This proposition says that the rate of employment can also respond positively to the aggregate technology shock A simultaneously with N if the supply of hours is sufficiently elastic (ε_w small). The intuition is that a higher aggregate technology raises the low-skilled workers' productivity, resulting in a higher rate of employment for the low-skilled workers, provided that the corresponding increase in the uniform working hours is not too big to curtail the positive effect of technology on the employment rate. This would be the case if the inverse labor supply curve is sufficiently flat or α is sufficiently large so that the cut-off function I is not too sensitive to changes in hours.

4 A Specific Example

Consider a parameterized model economy. Let N be the uniform working hours, and let the per-worker production function be given by $y_i = Ap_i N^\alpha$, where A represents an aggregate productivity shifter. In addition, let the productivity parameter of individual i follow $p_i = 1 - i, i \in [0, 1]$, and the inverse labor supply function be given by

$$w = \gamma_0 + \gamma_1 N. \tag{31}$$

Suppose the cut-off worker type is I, then workers with $i \leq I$ will be employed at wage rates $w_i = \alpha A p_i N^{\alpha-1}$, and workers with $i > I$ will be unemployed. Given N, the cut-off point I is determined by the condition:

$$\alpha A(1 - I)N^{\alpha-1} = \gamma_0 + \gamma_1 N, \tag{32}$$

or

$$I = 1 - \frac{(\gamma_0 + \gamma_1 N)}{\alpha A} N^{1-\alpha}. \tag{33}$$

Each employed worker ($i \leq I$) receives the real wage $w_i = \alpha A(1-i)N^{\alpha-1}$, which is greater than the cut-off worker's real wage by a factor of $\frac{1-i}{1-I} \geq 1$.

The firm chooses synchronized working hours N to solve

$$\max_N \int_{i=0}^{i=I(N)} (1-\alpha)AN^\alpha(1-i)di = (1-\alpha)AN^\alpha(I - \frac{1}{2}I^2), \tag{34}$$

where I is given in (33). The first order condition is

$$\alpha = (\varepsilon_w(N) + 1 - \alpha)\,\frac{(1-I)^2}{I(1-0.5I)}, \tag{35}$$

where $\varepsilon_w > 0$ is the elasticity of wage with respect to hours supply: $\frac{w'N}{w}$.

An implicit solution for the equilibrium rate of employment, I, is given by

$$I = 1 - \sqrt{\frac{\alpha}{2\varepsilon_w(N) + 2 - \alpha}}. \tag{36}$$

The solution is implicit because the wage elasticity, $\varepsilon_w(N)$, still depends on equilibrium hours worked N:

$$\varepsilon_w = \frac{\gamma_1 N}{\gamma_0 + \gamma_1 N}. \tag{37}$$

It is easy to see, however, that the rate of employment (I) and hours worked (N) comove in response to aggregate technology shocks A, regardless of α. Differentiating both sides of equation (36) with respect to the aggregate technology shifter A, we get:

$$\frac{dI}{dA} = \eta\varepsilon_w'(N)\frac{dN}{dA}, \tag{38}$$

where $\eta \equiv \sqrt{\frac{\alpha}{(2\varepsilon_w+2-\alpha)^3}} > 0$ and $\varepsilon_w'(N) = \frac{\gamma_0}{(\gamma_0+\gamma_1 N)^2} > 0$. Hence the direction of changes in I is the same as the direction of changes in N regardless of α. For this reason, we can assume $\alpha = 1$ without loss of generality, so as to gain further insight on the comovement of I and N. When $\alpha = 1$, the solutions for I and N are simple and explicit:[13]

$$I = 1 - \frac{\gamma_0 + \sqrt{\gamma_0^2 + 3A^2}}{3A}, \quad \gamma_0 \leq A \tag{39}$$

$$N = \frac{-2\gamma_0 + \sqrt{\gamma_0^2 + 3A^2}}{3\gamma_1}, \quad \gamma_0 \leq A. \tag{40}$$

Differentiating both equations with respect to A gives

$$\frac{\partial I}{\partial A} = \frac{\gamma_0\sqrt{\gamma_0^2 + 3A^2} + \gamma_0^2}{3A^2\sqrt{\gamma_0^2 + 3A^2}} > 0, \tag{41}$$

and

$$\frac{\partial N}{\partial A} = \frac{A}{\gamma_1\sqrt{\gamma_0^2 + 3A^2}} > 0. \tag{42}$$

It is a well documented stylized fact in the business cycle literature that both the rate of employment and hours worked are procyclical (e.g., see Cho and Cooley, 1994). This is consistent with the predictions of our model. A lower period of aggregate productivity induces profit-seeking firms to adjust downward both the number of employees and the number of hours worked per person. Since it is the low-skilled workers who are exposed to the layoff risk when employment rate decreases, the unemployment rate of low skilled workers is therefore more sensitive to the business cycle than that of skilled workers.

The relative magnitude of adjustment in the two different margins (number of workers and number of hours) in response to aggregate disturbances depend crucially on the slope of

[13] The firm's profit is zero when the technology is linear. In that case, the total revenue rather than the total profit is being maximized.

the labor supply curve and on the magnitude of the disturbance itself:

$$\frac{\partial I/\partial A}{\partial N/\partial A} = \gamma_1 \left(\frac{\gamma_0\sqrt{\gamma_0^2 + 3A^2} + \gamma_0^2}{3A^3} \right). \tag{43}$$

A flatter labor supply curve (a larger γ_1) or a lower propensity to work (a larger γ_0) implies more volatile employment rate relative to hours worked during the business cycle. Given the status quo of the labor supply curve, however, a lower level of aggregate productivity implies relatively smaller reactions from hours worked to business cycle shocks than that from employment rate. This prediction is interesting as it indicates that developed economies would have a higher volatility in hours worked but a lower volatility in employment rate than underdeveloped economies, as the impact of technology shocks would be mostly absorbed by hours worked in economies where aggregate productivity level is high.

The Case of Indivisible Labor – In the above discussions, we have considered synchronized working schedule due solely to cost minimization from the firm side, but synchronization of working hours can also be due to biological reasons (e.g., it is only natural for the human body to sleep during the night and work during the day). Suppose that for biological reasons only two discrete choices of hours exist: either working for $\bar{N}$ hours or not working at all. What are the consequences of indivisible labor on employment when workers are heterogenous in their skill levels?

Assuming $\alpha = 1$ for simplicity, the firm's profit-maximization program then becomes

$$\max_N \ \Pi = AN \int_{i=0}^{i=I(N)} p_i di \tag{44}$$

subject to

$$N = \{0, \hat{N}\}. \tag{45}$$

The solution is obviously $N = \hat{N}$, since the profit is zero when $N = 0$. Hence equation (20) or (33) suffices for determining the equilibrium level of employment in the model, which is

$$I = \max\left(0, 1 - \frac{\gamma_0}{A} - \frac{\gamma_1}{A}\hat{N}\right), \quad \gamma_0 < A. \tag{46}$$

Note that both the aggregate technology (A) and the length of working hours ($\hat{N}$) affect the equilibrium rate of employment. Since hours are indivisible, the adjustment of output in response to aggregate technology shock (A) falls entirely upon the rate of employment I. In particular, the employment rate decreases as A decreases. Again, in this model, unemployment falls upon the low-skilled workers because of the synchronization of labor. Obviously, any model with the assumption of indivisible labor hours cannot explain why hours also respond to business cycle disturbances. It is hence more likely that biological factors, institutional factors, and production coordination all play a role in synchronizing people's working schedules. For example, biological or institutional factors allow people to work for 8 hours per day and 40 hours per week on average, but for reasons of production coordination and profit maximization, firms can adjust the actual working hours up or down (say between 35 and 45 hours per week) in response to business conditions.

5 Robustness

In this section, we prove that explicitly taking into account workers' utility function in determining their labor supply behavior does not alter the conclusions reached in this paper, as long as the utility function is consistent with an upward-sloping labor supply curve (i.e., the substitution effect dominates the income effect). The crucial thing to check is that such considerations do not affect the main features of the cut-off function (I) which were determined previously by the condition:

$$p_I f'(N) = w_I(N), \tag{47}$$

where the right hand side is the real wage determined by the marginal product, and the left-hand side is the worker's inverse labor supply function. There are two major properties implied by this condition which were used previously to prove propositions (2)-(5). The first property is that $\frac{dp_I}{dN} > 0$, which also implies $\frac{dI}{dN} < 0$ since p_I is decreasing in I. The second property is that the cumulative productivity index, $P \equiv \int_{i=0}^{I(N)} p_i di$, is decreasing in N : $\frac{dP}{dN} > 0$. This is a natural consequence of $\frac{dI}{dN} < 0$.

Let the index number (employment rate) that solves condition (47) be I_1. The assumption behind condition (47) for being a marginal condition is that workers are better off by not working than working for more hours than desired. We show here that relaxing this assumption does not change anything qualitatively except that the newly determine cut-off point (call it I_2) lies above I_1, the original cut-off point determined from (47). The intuition for $I_2 > I_1$ is that workers with indices immediately above I_1 (i.e., with labor demand curves lie immediately below worker I_1) may also find working for N hours more attractive than not working at all, although N exceeds their desired labor supply. Workers with indices $i > I_2$, however, definitely find working not attractive as the utility received from working for N hours is less than that from not working at all.

Consider the utility function for worker $i \in [0, 1]$:

$$u(c_i, 1 - n_i) = u(\gamma + w_i n_i, 1 - n_i), \quad u_1', u_2' > 0, u_1'', u_2'' < 0; \tag{48}$$

where $c_i = \gamma + w_i n_i$ is consumption, w_i is the real wage, n_i is the number of hours worked (the time endowment has been normalize to 1), and $\gamma > 0$ is the wealth level. The optimal labor supply is determined by

$$u_c(c_i, n_i)\frac{dc}{dn} = u_{1-n}(c_i, n_i), \tag{49}$$

or

$$u_c\left(\gamma + w_i n_i, 1-n\right) w = u_{1-n}\left(\gamma + w_i n_i, 1-n\right). \tag{50}$$

Let $w_i = p_i f'(n_i)$, $i = I_1$, and $n_i = N$, (50) becomes exactly the condition (47) which was used to determine the cut-off point I_1 previously in the paper, provided that the inverse labor supply function $w(n_i)$ as an implicit solution to (50) exists and is unique. But (50) is no longer the right condition for determining the cut-off point in the current case, as some workers with $i > I_1$ may also prefer working to not working. The right condition is given by

$$u\left(\gamma + p_i f'(N)N, 1-N\right) \geq u(\gamma, 1). \tag{51}$$

Namely, facing the synchronized working hours N, the individual will choose to work if and only if the utility received from working for N hours exceeds the utility of not working at all. Hence, a cut-off point $I_2(N)$ exists and is determined implicitly by the equation,

$$u\left(\gamma + p_{I_2} f'(N)N, 1-N\right) = u(\gamma, 1). \tag{52}$$

Note that the following inequalities hold:

$$u_c(c_i, N)\frac{dc}{dN} > u_{1-n}(c_i, N), \quad \text{if } i \leq I_1, \tag{53}$$

$$u_c(c_i, N)\frac{dc}{dN} < u_{1-n}(c, N), \quad \text{if } i > I_1; \tag{54}$$

namely, for a worker i ($\leq I_1$) whose desired labor supply is greater than that of the marginal worker I_1 which is determined by (50), increasing her working hours beyond N increases her utility; and for a worker ($i > I_1$) whose desired labor supply is less than that of the marginal worker I_1, increasing her working hours beyond N decreases her utility.

Given that $I_2 > I_1$, we thus have

$$u_c(c_{I_2}, N)\frac{dc}{dN} < u_{1-N}(c_{I_2}, N). \tag{55}$$

Now totally differentiating the cut-off equation (52) with respect to N, realizing that $I = I(N)$ and $f'(N)N = \alpha f(N)$, gives

$$u_c\left(c_{I_2}, N\right)\left(\frac{dp_{I_2}}{dN}\alpha f(N) + \frac{dc}{dN}\right) = u_{1-N}\left(c_{I_2}, N\right). \tag{56}$$

Comparing (56) with (55) immediately gives

$$\frac{dp_{I_2}}{dN} > 0. \tag{57}$$

Since p_I is decreasing in I, we also have

$$\frac{dI_2}{dN} < 0. \tag{58}$$

This completes the proof.

Figure 3 illustrates the idea fully, where the rays represent budget constraints for different types with different wage income, and the convex curves represents indifference curves. Note that agent I_1 has desired labor supply just equal to her labor demand at N, but she is no longer the cut-off type. The cut-off type (or the employment rate) is determined instead by agent I_2 who is just indifferent between working for N hours and not working at all.

6 Conclusion

In this paper, a simple model is proposed to explain certain forms of structural unemployment, without resorting to conventional labor-market frictions such as sticky wages or imperfect information on workers' productivity. The theory is built on two commonly observed facts. First, working schedules are highly synchronized across labor. For example, in the 1890s, about 47% of male workers in the U.S. worked 10 hours per day and "the most common pattern was for work to begin at 7:00 A.M. and end at 5:30 P.M. with a 30-minute break for lunch" (Costa 2000, p159). One hundred years later, the degree of synchronization had become even stronger. In 1991, 57% of male workers in the US. reported that they worked 8 hours per day, and the most common pattern was from 8:00 A.M. to 5:00 P.M. (Costa 2000, p160). The second observation is that workers are heterogeneous in their skill levels, some being more productive than others (this is true even for workers in the same department working on similar tasks). Differences in productivity imply differences in wages, which in turn imply differences in hours supply. Synchronization of labor, however, requires the same length of working hours. As a result, low-skilled workers are more likely to be unemployed than skilled workers, given similar propensities to work, since firms cannot afford to pay low-skilled workers their reservation wages, which are above their marginal products.

Our analysis also shows that the rate of employment and the average hours worked per worker can both respond to business cycle shocks in the same direction. During economic booms, not only the average working hours (synchronized across workers) are longer, but more people are also absorbed into the work force from the low end of the skill spectrum.

The converse is also true during economic recessions. As a result, low-skilled workers are more sensitive to the business cycle than skilled workers, as is observed in the US economy.

Since our model predicts that the rate of employment depends negatively on the length of working hours, other things equal, an obvious policy implication of the model is this: reducing the length of working hours or offering part-time jobs can boost employment. The reason is that more low-skilled workers are able to find jobs when the working time shortens. The French government, for example, has been pushing for a 35-hour work week against the traditional 40-hour work week in an attempt to reduce unemployment. The welfare gain of such policy, however, is not clear, since a shorter working time also causes a loss of aggregate output as low-skilled labor hours replace high-skilled labor hours. Thus the situation depends on the balance between the gains of output due to a higher rate of employment and the loss of output due to a shorter working time.[14] In future works, we hope to push for a carefully evaluation of the welfare consequence of adopting shorter work week in an environment like the one we have modeled here.

[14] In addition, the high-skilled workers are worse off because they would rather work more hours at the given wage than fewer.

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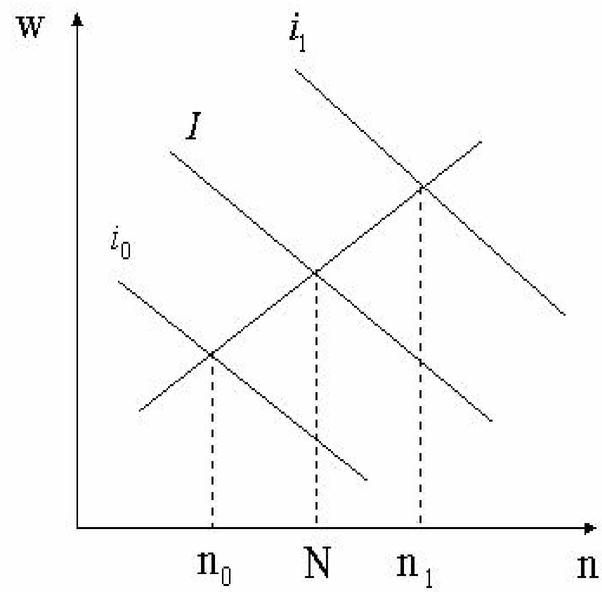

Figure 1. Labor market equilibrium when hours are not synchronized, where agent i_0 works for n_0 hours, agent I works for N hours, and agent i_1 works for n_1 hours.

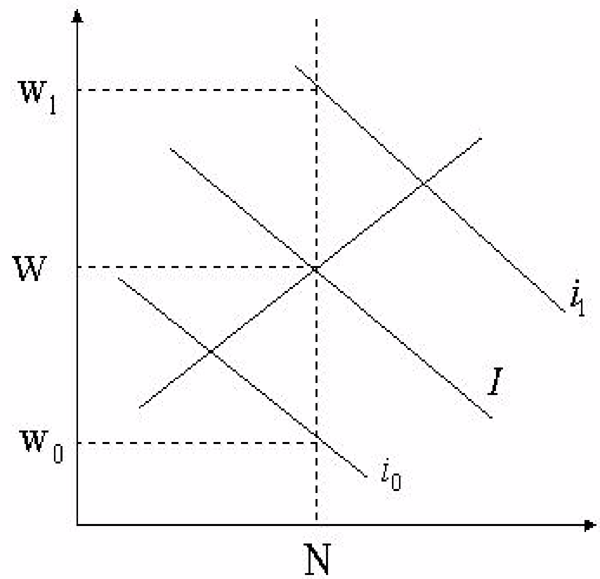

Figure 2. Labor market equilibrium when hours are synchronized at N, where worker i_0 is unemployed, worker i_1 is employed, and worker I is the cut-off type.

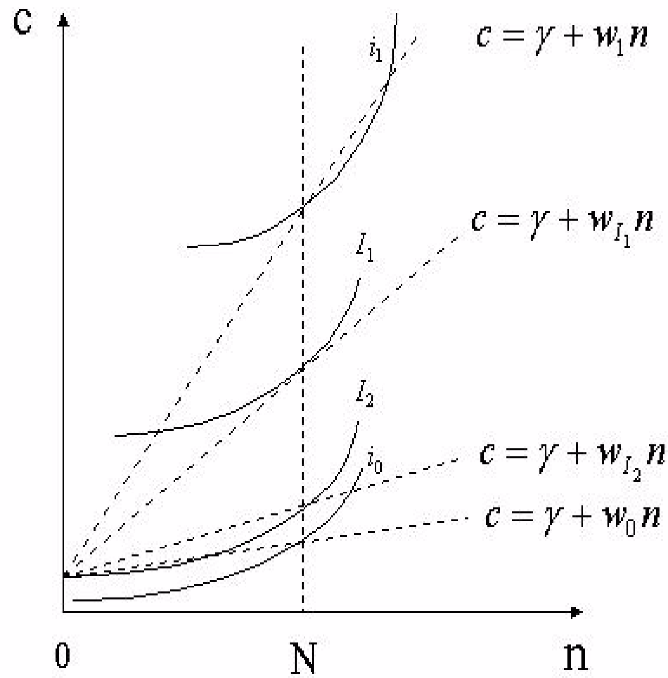

Figure 3. Labor market equilibrium when hours are synchronized, where worker i_0 is unemployed, workers I_1 and i_1 are employed, and worker I_2 is the cut-off type.